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Stringy and Membranic Theory of Swimming of Micro-organisms *

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Abstract

When the swimming of micro-organisms is viewed from the string and membrane theories coupled to the velocity field of the fluid, a number of interesting results are derived; 1) importance of the area (or volume) preserving algebra, 2) usefulness of the N -point Reggeon (membranic) amplitudes, and of the gas to liquid transition in case of the red tide issues, 3) close relation between the red tide issue and the generation of Einstein gravity, and 4) possible understanding of the three different swimming ways of micro-organisms from the singularity structure of the shape space.

We are very happy to present our recent works on the swimming of micro-organisms at Asia Pacific Conference on Gravitation and Cosmology held at Sheraton Walker Hill Hotel in Seoul. The talk will be based mainly on Refs.[1]-[4]. Amazing is the fact that there are only three kinds of swimming ways of micro-organisms; (1) Ciliated motion for the Paramecium etc. using cilia (small hairs covering its surface), (2) flagellated motion for the sperm etc. using the flagellum (whip), and (3) the swimming of bacteria with the so-called bacterial flagella (whips moving like the wine-opener). Why does such a simple classification exists? That is a very difficult issue, but we will challenge it. Our strategy is to consider the boundary of the micro-organisms as the strings or membranes $X^\mu(t; \xi)$ with the parametrization ξ , whose motion is coupled with the external fluid dynamics.

1. The fluid dynamics of the micro-organisms is very simple, since the small size $L \ll 1\text{mm}$ of the micro-organisms implies the Reynolds number $R \ll 1$ and we have the following equations of motion for the velocity field $v^\mu(x)$ and the pressure $p(x)$: $\Delta v_\mu = \partial_\mu p / \mu$ or $\Delta(\partial_\mu v_\nu - \partial_\nu v_\mu) = 0$, where μ is the coefficient of viscosity.

*The invited talk given by A. Sugamoto

The deformation of the boundary by the micro-organisms themselves should be equal to the fluid velocity there: $v^\mu(\vec{X}(t;\xi)) = \dot{X}^\mu(t;\xi)$. These simple equations can determine the swimming motion of the micro-organisms [5]: Deformation of the micro-organisms by their own ways may cause unwanted flows at spacial infinity. Then the swimming motion is so determined as to cancel these flows.

2. We have analyzed flagellated as well as the ciliated motions in the spacial dimensions $D = 2$, perturbatively with the small deformations $\alpha(t, \theta)$, where θ is the parametrization of the circle (the boundary of the ciliate) and the circle obtained by the Joukowski transform from the line (the boundary of the flagellate). We have selection rules, i.e., the relation between two required Fourier mode-numbers, n_1 and n_2 , in order for the micro-organisms to swim (see Ref.[1]).
3. The boundaries of the ciliate and the flagellate for $D = 2$ are the closed and the open strings, respectively. The selection rules, however, differ from the mode number relations in the translation and rotation operators (belonging to the Virasoro algebra) of the string theories. To understand the discrepancy, it may be helpful to study the area (volume) preserving diffeomorphisms, since the incompressibility of the fluid implies these. They are the $w_{1+\infty}$ algebra and its generalization for $D = 2$, the representation theory of which as well as of their quantum version are well studied recently. Quantum version gives the uncertainty principle between the complex coordinate z and its canonical conjugate \bar{z} . The Planck constant \hbar in our case can be the thermal fluctuation squared of the position, namely, $\sim (\Delta z)^2$. In the near future a clever application of the representation theory of these algebras will classify the swimming ways of micro-organisms.
4. The action of the N -body micro-organisms coupled with the fluid can be written as follows:

$$\begin{aligned} \hat{S}_N = & \sum_{i=1}^N \int dt \int d^{D-1} \xi_{(i)} P_\mu^{(i)}(t; \xi_{(i)}) \left[\dot{X}_\mu^{(i)}(t; \xi_{(i)}) - v^\mu(X_{(i)}) \right] \\ & - \frac{1}{2\pi\alpha'} \int d^D x \left[\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{\mu} p(x) \partial_\mu v^\mu \right], \end{aligned}$$

where $\omega_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$ is the vorticity, and $P_\mu^{(i)}$ is the Lagrange multiplier. The system is nothing but the Landau gauge QED with the velocity field v^μ as a gauge field. On the other hand ‘‘Landau-Lifshitz hydrodynamics’’ tells us that the temporal growth of the entropy \dot{S} can be given by

$$\dot{S} = \int d^D x \left[\frac{\mu}{2T} (\partial_\nu v_\lambda + \partial_\lambda v_\nu)^2 + \mathbf{q} \cdot \nabla \left(\frac{1}{T} \right) \right],$$

where \mathbf{q} is the heat current. The path integral weight given by the action and the statistical weight given by the entropy coincides if the Regge slope α' satisfies

$\frac{1}{4\pi\alpha'} = \frac{\mu t^*}{k_B T}$, where the t^* is the typical time scale or the period of the swimming motion [3].

5. Probability G_N of having the given positions $X_{(1)}, \dots, X_{(N)}$ and the given deformation velocities $\dot{X}_{(1)}, \dots, \dot{X}_{(N)}$ of the N micro-organisms, can be estimated with the path integral weight just discussed above, leading to [1]

$$\begin{aligned} G_N(X_{(1)}, \dot{X}_{(1)}; \dots; X_{(N)}, \dot{X}_{(N)}) \\ = \int \mathcal{D}P_\mu^{(i)} \exp \left\{ i \sum_{i=1}^N \int dt \int d^{D-1} \xi_{(i)} P_\mu^{(i)} \dot{X}^\mu \right\} \\ \times \exp \left[2\pi i \alpha' \times \frac{1}{2} \sum_{i,j} \int dt_{(i)} \int d^{D-1} \xi_{(i)} \int dt_{(j)} \int d^{D-1} \xi_{(j)} \right. \\ \left. \times P_\mu^{(i)}(t_{(i)}; \xi_{(i)}) G_\perp^{\mu\nu} (X_{(i)}(\xi_{(i)}) - X_{(j)}(\xi_{(j)})) P_\mu^{(j)}(t_{(j)}; \xi_{(j)}) \right]. \end{aligned}$$

The result is the N -point Reggeon for $D = 2$ (field theoretical membrane for $D = 3$) amplitude with the transverse Green's functions G_\perp . In the terminology of the string or the membrane theory, v^μ belongs to the target space whereas the coordinates x^1, \dots, x^D belong to the parameter space. Therefore the existence of the handles may corresponds to those of the vortices. If $\vec{X}_N \rightarrow \infty$, $-\vec{X}_N$ represents the collective motion of $N - 1$ micro-organisms and G_N is its probability, so that the averaged collectively swimming velocity reads

$$\vec{V}_{N-1} = -\langle \dot{X}_N \rangle = - \sum_{\dot{X}_N} \dot{X}_N G_N(X_1, \dot{X}_1; \dots; X_N, \dot{X}_N).$$

6. In summer, the abnormally generated plankton (micro-organism) forms the red tide. It behaves like a fluid (we gave the illustration picture of the red tide at the conference.). To understand the gas to liquid transition according to the increase of the number density of the plankton, we have to show the van der Waals type equation of state,

$$\left(p + \frac{N^2}{V^2} (a + a'/k_B T + \dots) \right) (V - Nb) = N k_B T,$$

where terms with the coefficients a, a', \dots represents the 2 body attractive forces between atoms (micro-organisms), whereas the b represents the occupied volume of the atom (micro-organism). To estimate the coefficients a, a', \dots and b we extract the interaction potential $\phi(x)$ from the action and calculate the so-called Meyer integral. Then we have

$$\int d^D x \left(e^{-\phi(x)/k_B T} - 1 \right) = -2b + 2a(k_B T)^{-1} + 2a'(k_B T)^{-2} + \dots$$

Vanishing of a occurs by the angle averaging of the multipole type potential $\phi(x) \sim \cos(2\ell\theta + \alpha)/r^{-2\ell-2}$ ($\ell = -2, -3, \dots$) between two micro-organisms with the relative

coordinates (r, θ) and the same oscillation mode ℓ . Now we find that the gas to liquid transition occurs when the number density n of the plankton increases as $n \geq 10$ plankton/m³ for the 0.1 mm plankton in the water at temperature $T \sim 300\text{K}$. The result is not so unrealistic!? [3]

7. As a similar phenomenon to the condensation of micro-organisms (red tide), we can consider that the condensation of strings may trigger the phase transition of the 2-form gravity to the Einstein gravity. The 2-form gravity has the extra Kalb-Ramond symmetry which is, however, broken by the ansatz of introducing the metric. Instead, we write down the Kalb-Ramond invariant action by introducing the extra string field whose condensation leads to the ansatz [7]. This is a kind of the Meissner effect in the string theory. To understand the gravity from the swimming of microorganisms it is worth remembering that 20 years ago Professor Nambu called Kalb-Ramond theory the relativistic hydrodynamics.
8. Finally we come to the recent work [4] by one of the authors (Masako Kawamura). She has calculated for the flagellate the efficiency η of the micro-organism's swimming *à la* Shapere and Wilczek. Her result is $\eta = \frac{t^*}{16\pi\mu} \frac{1}{|\sin\beta'|}$ which can be unboundedly large for $\beta' \rightarrow 0$, whereas the efficiency of the ciliated swimming is bounded from above as $\eta = \frac{\sqrt{5}t^*}{8\pi^2\mu} |\sin\beta|$. Therefore the flagellated swimming is much more efficient than the ciliated. This might be a reason why the size of the flagellate can be smaller than the ciliate. Furthermore, this gives a very interesting indication that in the shape space (in which each point represents a different shape of the micro-organism), there exists the conical singularity at the point of the flagellate. This can be understood as follows: One cycle of the swimming motion draws a closed path in the shape space and the efficiency is the period times the ratio of the area inside the closed path over its perimeter squared.
9. Conclusion at Seoul:
 - (a) Swimming of micro-organisms can be viewed from string and membrane theories, coupled to the velocity field of the fluid.
 - (b) In the future, area (or volume) preserving algebra ($w_{1+\infty}$ or $W_{1+\infty}$) may help the understanding of micro-organisms' swimming.
 - (c) Collective motion of micro-organisms can be studied by the N -point Reggeon (membranic) amplitudes, or the gas to liquid transition (red tide). Essentially, the understanding of the interaction between strings and membranes is important [8, 9]. This is also intimately related to the string or membrane's condensation and the generation of Einstein gravity from the topological one.
 - (d) From the estimation of the swimming efficiency, difference between the ciliate and the flagellate comes out. In this respect the classification of the three kinds

of swimming ways can be understood from the geometry of the shape space or its singularity structure.

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